

# Review 11 Related Rates

big idea: given one rate, find another rate

rate is derivative wrt time  $\frac{units}{\frac{t}{sec}}$   $\frac{m}{sec}$   $\frac{gal}{hr}$

look for key words like:

"How fast?", "velocity", "increasing", "decreasing"

1. Understand the problem  
draw pic, label variables, constants
2. write an equation  
derivatives
3. take derivative wrt "t"
4. plug in stuff
5. solve for the unknown der.

Ex 1 <sup>cylindrical</sup> A tank has a radius of 5 ft. water leaks out at a rate given by  $\frac{dv}{dt} = -5\pi\sqrt{h}$   $\frac{ft^3}{min}$   
show the depth changes at a rate of  $-\frac{\sqrt{h}}{5}$   $\frac{ft}{min}$



$$\frac{dv}{dt} = -5\pi\sqrt{h}$$

$$V = \pi r^2 h$$

$$V = \pi \cdot 5^2 h$$

$$\frac{dv}{dt} = 25\pi \frac{dh}{dt}$$

$$-5\pi\sqrt{h} = 25\pi \frac{dh}{dt}$$

$$-\frac{\sqrt{h}}{5} = \frac{-5\pi\sqrt{h}}{25\pi} = \frac{dh}{dt}$$

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Ex 2. Water pours into a spherical balloon at a rate of  $6 \frac{in^3}{sec}$ . How fast is the radius changing when it is 2 in?

$$6 \frac{in^3}{sec} = \frac{dv}{dt}$$

$v$  = volume  $r$  = radius



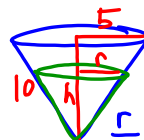
$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{6}{16\pi} \frac{in}{sec}$$

$$6 = 4\pi \cdot 2^2 \frac{dr}{dt}$$

Ex 3 water pours into a conical tank at a rate of  $9 \frac{ft^3}{min}$ . The tank has a radius of 5 ft and a height of 10 ft. How fast is the water level rising when ~~the~~ <sup>the water level</sup> is 6 ft?



$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{r}{h} = \frac{5}{10}$$

$$r = \frac{h}{2}$$

$$\frac{dv}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$9 = \frac{\pi}{4} 6^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{9 \cdot 4}{\pi \cdot 6^2} = \frac{1}{\pi}$$

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